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Light quark masses from scalar sum rules

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Abstract. In this work, the mass of the strange quark is calculated from QCD sum rules for the divergence of the strangeness-changing vector current. The phenomenological scalar spectral function which enters the sum rule is determined from our previous work on strangeness-changing scalar form factors [1]. For the running strange mass in the $\overline{\text{MS}}$ scheme, we find $m_s(2\,\text{GeV}) = 99 \pm 16\,\text{MeV}$. Making use of this result and the light quark mass ratios obtained from chiral perturbation theory, we are also able to extract the masses of the lighter quarks m_u and m_d. We then obtain $m_u(2\,\text{GeV}) = 2.9 \pm 0.6\,\text{MeV}$ and $m_d(2\,\text{GeV}) = 5.2 \pm 0.9\,\text{MeV}$. In addition, we present an updated value for the light quark condensate.

1 Introduction

Together with the strong coupling constant, quark masses are fundamental QCD input parameters of the standard model, and thus their precise determination is of paramount importance for present day particle phenomenology. In the light quark sector, the mass of the strange quark m_s deserves particular interest, because its present uncertainty severely limits the precision of current predictions of the CP-violating observable ε'/ε. The ratios of light quark masses are known rather precisely from chiral perturbation theory χPT [2,3], and thus, once the absolute scale is set by m_s, also the masses of the lighter up and down quarks can be determined.

Until today, two main methods have been employed to determine the strange quark mass. QCD sum rules [4–7] have been applied to various channels containing strange quantum numbers, in particular the scalar channel that will be the subject of this work [8–15], the pseudoscalar channel [16], the Cabibbo suppressed τ-decay width [17–23], as well as the total e^+e^- cross section [24–27]. Also lower bounds on the strange mass have been determined in the framework of QCD sum rules [28–31,16]. In addition, lattice QCD simulations for various hadronic quantities have been used to extract the strange quark mass. For two recent reviews where original references can be found, the reader is referred to [32,33].

The dispersive QCD sum rule approach makes use of the phenomenological knowledge on the spectral functions associated with hadronic currents with the corresponding quantum numbers. From the experimental point of view at present the cleanest information comes from τ decays [19]; however, up to now the Cabibbo suppressed hadronic τ-decay data has not been resolved into separate $J = 0$ and $J = 1$ contributions and the theoretical uncertainties associated with a bad perturbative behaviour of its scalar component put a limit on the achievable accuracy [17–23].

The more standard analysis of the scalar or pseudoscalar currents provides a large sensitivity to light quark masses. Unfortunately, the rather large uncertainties of the $J = 0$ data introduce important systematic errors in the resulting quark mass determination, which are difficult to quantify. Previous analyses have used phenomenological parameterisations based on saturation by the lightest hadronic states with the given quantum numbers, sometimes improved with Breit–Wigner and/or Omnès expressions [8–16].

In two recent papers, we have presented very detailed analyses of S-wave $K\pi$ scattering [34] and the $K\pi$, $K\eta$ and $K\eta'$ scalar form factors [1], which incorporate the experimental knowledge on the $J = 0$ $K\pi$ phase shifts as well as all known theoretical constraints from chiral perturbation theory, short-distance QCD, dispersive relations, unitarity and large-N_c considerations. The output of these works is a rather reliable determination of the scalar spectral function up to about 2 GeV. This allows us to perform a considerable step forward in the QCD sum rule determination of light quark masses through the scalar correlators.

The central object which is investigated in the original version of QCD sum rules [4] is the two-point function $\Psi(p^2)$ of two hadronic currents

$$\Psi(p^2) \equiv \mathrm{i} \int \mathrm{d}x\, \mathrm{e}^{\mathrm{i}px} \langle \Omega | T\{ j(x) j(0)^\dagger \} | \Omega \rangle, \tag{1.1}$$

where Ω denotes the physical vacuum and in our case $j(x)$ will be the divergence of the strangeness-changing vector current,

* Heisenberg fellow

$$j(x)=\partial^\mu(\bar{s}\gamma_\mu q)(x)=\mathrm{i}(m_s-\hat{m})(\bar{s}q)(x). \qquad (1.2)$$

Since we work in the isospin limit, q can be either an up- or down-type quark, and $\hat{m}$ is the isospin average $\hat{m}=(m_u+m_d)/2$. To a good approximation, $\Psi(p^2)$ is thus given by m_s^2 times the two-point function of the scalar current.

Up to a subtraction polynomial, $\Psi(p^2)$ satisfies a dispersion relation,

$$\Psi(p^2)=\Psi(0)+p^2\Psi'(0)+p^4\int\limits_0^\infty\frac{\rho(s)}{s^2(s-p^2-\mathrm{i}0)}\mathrm{d}s, \qquad (1.3)$$

where $\rho(s)\equiv\mathrm{Im}\Psi(s+\mathrm{i}0)/\pi$ is the spectral function corresponding to $\Psi(s)$. To suppress contributions in the dispersion integral coming from high invariant-mass states, it is convenient to apply a Borel (inverse Laplace) transformation to (1.3) [4], which furthermore removes the subtractions. The left-hand side of the resulting equation is calculable in QCD, whereas under the assumption of quark–hadron duality, the right-hand side can be evaluated in a hadron-based picture, thereby relating hadronic quantities to the fundamental QCD parameters.

Generally, however, from experiments the phenomenological spectral function $\rho_{\mathrm{ph}}(s)$ is only known from threshold up to some energy s_0. Above this value, we shall use the perturbative expression $\rho_{\mathrm{th}}(s)$ also for the right-hand side. This is legitimate if s_0 is large enough so that perturbation theory is applicable. The central equation of our sum rule analysis for m_s is then

$$u\mathcal{B}_u[\Psi_{\mathrm{th}}(p^2)]\equiv u\widehat{\Psi}_{\mathrm{th}}(u)=\int\limits_0^{s_0}\rho_{\mathrm{ph}}(s)\mathrm{e}^{-s/u}\mathrm{d}s + \int\limits_{s_0}^\infty\rho_{\mathrm{th}}(s)\mathrm{e}^{-s/u}\mathrm{d}s, \qquad (1.4)$$

where $\mathcal{B}_u$ is the Borel operator, the hat denotes the Borel transformation, and u is the so-called Borel variable. The main ingredients in this equation, namely the theoretical expression for the two-point function as well as the phenomenological spectral function, will be discussed below[1].

In addition, it is instructive to investigate the sum rule which arises by considering the Borel transform of $\Psi(p^2)/p^2$:

$$u\mathcal{B}_u\left[\frac{1}{p^2}\Psi_{\mathrm{th}}(p^2)\right]\equiv\widehat{\Phi}_{\mathrm{th}}(u)=\int\limits_0^{s_0}\frac{\mathrm{d}s}{s}\rho_{\mathrm{ph}}(s)\mathrm{e}^{-s/u} + \int\limits_{s_0}^\infty\frac{\mathrm{d}s}{s}\rho_{\mathrm{th}}(s)\mathrm{e}^{-s/u}-\Psi(0). \qquad (1.5)$$

The sum rule (1.5) is constructed such that the subtraction constant $\Psi(0)$ remains. However, from a Ward identity [35] this constant is related to the following product of quark masses and quark condensates:

$$\Psi(0)\;=\;(m_s-\hat{m})(\langle\Omega|\bar{q}q|\Omega\rangle-\langle\Omega|\bar{s}s|\Omega\rangle). \qquad (1.6)$$

Note that the quark condensates in (1.6) appear as non-normal-ordered vacuum averages, and thus $\Psi(0)$ is not renormalisation group invariant [36,37,10,38]. The corresponding renormalisation invariant quantity involves additional quartic quark mass terms [36]. Because of the dependence on $\Psi(0)$, analysing the sum rule of (1.5) would enable us to obtain information on the quark condensates. As we shall show in the next section, however, the perturbative expansion for $\widehat{\Phi}_{\mathrm{th}}(u)$ behaves very badly, and thus such an analysis appears to be questionable. Additional discussion of $\Psi(0)$ can also be found in [38].

In the next two sections, we present expressions for the theoretical as well as phenomenological two-point functions which are relevant for the sum rules under investigation. In Sect. 4, we then discuss the numerical analysis of the strange mass sum rule. Finally, in our conclusions, we compare our results with other recent determinations of m_s, calculate the light quark masses m_u and m_d from mass ratios known from χPT, and update our current knowledge of the quark condensate.

2 Theoretical two-point function

In the framework of the operator product expansion the Borel transformed two-point function $\widehat{\Psi}_{\mathrm{th}}(u)$ can be expanded in inverse powers of the Borel variable u:

$$\widehat{\Psi}_{\mathrm{th}}(u)=(m_s-\hat{m})^2u \times\left[\widehat{\Psi}_0(u)+\frac{\widehat{\Psi}_2(u)}{u}+\frac{\widehat{\Psi}_4(u)}{u^2}+\frac{\widehat{\Psi}_6(u)}{u^3}\right]. \qquad (2.1)$$

The $\widehat{\Psi}_n(u)$ contain operators of dimension n, and their remaining u dependence is only logarithmic. Below, we shall review explicit expressions for the first two of these contributions.

The purely perturbative contribution $\widehat{\Psi}_0(u)$ is presently known up to $\mathcal{O}(\alpha_{\mathrm{s}}^3)$ [39–41] and the expansion in the strong coupling up to this order reads

$$\begin{aligned}\widehat{\Psi}_0(u)=&\frac{3}{8\pi^2}\Bigg[1+\left(\frac{11}{3}+2\gamma_{\mathrm{E}}\right)a\\&+\left(\frac{5071}{144}-\frac{17}{24}(\pi^2-6\gamma_{\mathrm{E}}^2)+\frac{139}{6}\gamma_{\mathrm{E}}-\frac{35}{2}\zeta_3\right)a^2\\&+\Bigg(\frac{1995097}{5184}-\frac{\pi^4}{36}-\frac{695}{48}(\pi^2-6\gamma_{\mathrm{E}}^2)\\&-\frac{221}{48}\gamma_{\mathrm{E}}(\pi^2-2\gamma_{\mathrm{E}}^2)+\frac{2720}{9}\gamma_{\mathrm{E}}-\frac{475}{4}\gamma_{\mathrm{E}}\zeta_3-\frac{61891}{216}\zeta_3\\&+\frac{715}{12}\zeta_5\Bigg)a^3\Bigg]\\=&\frac{3}{8\pi^2}[1+1.535\alpha_{\mathrm{s}}+2.227\alpha_{\mathrm{s}}^2+1.714\alpha_{\mathrm{s}}^3], \qquad (2.2)\end{aligned}$$

where $a\equiv\alpha_{\mathrm{s}}/\pi$, γ_{E} is Euler's constant and $\zeta_z\equiv\zeta(z)$ is the Riemann ζ-function. In this expression the logarithmic corrections have been resummed to all orders, and

[1] Further details on the approach can for example be found in [9]

thus the strong coupling $\alpha_s(u)$ should be evaluated at the scale u. Higher order terms are also known in the large-N_f expansion [42] and partial results are known at order α_s^4 [43]. Even for $\alpha_s(1\,\text{GeV}) \approx 0.5$ the last term in (2.2) is only about 20% and the perturbative expansion displays a reasonable convergence. Because the two-point function scales as m_s^2, the resulting uncertainty for m_s from higher orders is at most 10%. In practice it is much smaller since the average scale at which the sum rule is evaluated lies around 1.5 GeV.

The theoretical two-point function $\widehat{\Phi}_{\rm th}(u)$ of (1.5) has an operator product expansion which is completely equivalent to (2.1), and the corresponding perturbative contribution takes the form

$$\begin{aligned}\widehat{\Phi}_0(u) &= \frac{3}{8\pi^2}\left[1+\left(\frac{17}{3}+2\gamma_{\rm E}\right)a\right.\\ &+\left(\frac{9631}{144}-\frac{17}{24}(\pi^2-6\gamma_{\rm E}^2)+\frac{95}{3}\gamma_{\rm E}-\frac{35}{2}\zeta_3\right)a^2\\ &+\left(\frac{4748953}{5184}-\frac{\pi^4}{36}-\frac{229}{12}(\pi^2-6\gamma_{\rm E}^2)\right.\\ &-\frac{221}{48}\gamma_{\rm E}(\pi^2-2\gamma_{\rm E}^2)+\frac{4781}{9}\gamma_{\rm E}-\frac{475}{4}\gamma_{\rm E}\zeta_3-\frac{87541}{216}\zeta_3\\ &\left.\left.+\frac{715}{12}\zeta_5\right)a^3\right]\\ &= \frac{3}{8\pi^2}[1+2.171\alpha_s+5.932\alpha_s^2+17.337\alpha_s^3]. \qquad (2.3)\end{aligned}$$

As is obvious from this expression, the perturbative expansion for $\widehat{\Phi}_0(u)$ behaves very badly. Even at a scale $u^{1/2} = 2\,\text{GeV}$, the last two terms are of comparable size and individually both are larger than 50% of the leading term. If the logarithmic corrections are not resummed, the perturbative expansion could be improved by taking a fixed scale μ. This would shift part of the corrections into the prefactor $(m_s - \hat{m})^2$. In this case one finds, however, that for $u^{1/2}$ in the range 1–2 GeV a reasonable size of the higher orders is only obtained if μ is much less than 1 GeV. But then the perturbative contribution is again questionable. To conclude, the huge perturbative corrections for $\widehat{\Phi}_0(u)$ prevent us from performing a sum rule analysis of (1.5).

The next term in the operator product expansion $\widehat{\Psi}_2(u)$ only receives contributions proportional to the quark masses squared. Its explicit expression reads

$$\begin{aligned}\widehat{\Psi}_2(u) = -\frac{3}{4\pi^2}&\left\{\left[1+\frac{4}{3}(4+3\gamma_{\rm E})a\right](m_s^2+m_u^2)\right.\\ &\left.+\left[1+\frac{4}{3}(7+3\gamma_{\rm E})a\right]m_s m_u\right\}. \qquad (2.4)\end{aligned}$$

Already at a scale of $u = 1\,\text{GeV}^2$ the size of $\widehat{\Psi}_2$ is less than 3%, decreasing like $1/u$ for higher scales. Although it has been included in the phenomenological analysis, for the error estimates on the strange quark mass it can be safely neglected.

The same holds true for the dimension-four operators. In this case there are contributions from the quark and gluon condensates as well as quark mass corrections of order m^4. Again, at a scale of $u = 1\,\text{GeV}^2$ the size of $\widehat{\Psi}_4$ is well below 1% of the full two-point function, hence being negligible for the strange mass analysis. Nevertheless, the dimension-four and in addition also the dimension-six contributions $\widehat{\Psi}_4$ and $\widehat{\Psi}_6$ have been included in our numerical investigations. Analytic expressions for these contributions are collected in Sect. 2 of [9].

To calculate the perturbative continuum on the right-hand side of (1.4), we also need the theoretical spectral function $\rho_{\rm th}(s)$ which is given by

$$\begin{aligned}\rho_{\rm th}(s) &= \frac{3}{8\pi^2}(m_s-\hat{m})^2 s \qquad (2.5)\\ &\times\left[1+\frac{17}{3}a+\left(\frac{9631}{144}-\frac{35}{2}\zeta_3-\frac{17}{12}\pi^2\right)a^2\right.\\ &\left.+\left(\frac{4748953}{5184}-\frac{91519}{216}\zeta_3+\frac{715}{12}\zeta_5-\frac{229}{6}\pi^2-\frac{\pi^4}{36}\right)a^3\right]\\ &= \frac{3}{8\pi^2}(m_s-\hat{m})^2 s[1+1.804\alpha_s+3.228\alpha_s^2+2.875\alpha_s^3].\end{aligned}$$

Again, the logarithms have been resummed, so that the coupling and masses are running quantities evaluated at the scale s. It is possible to calculate the relevant integral from s_0 to infinity in (1.4) analytically. The corresponding theoretical expressions can be found in [9].

3 Hadronic spectral function

The phenomenological spectral function is obtained by inserting a complete set of intermediate states Γ with the correct quantum numbers in the current product of (1.1),

$$\rho_{\rm ph}(s) = (2\pi)^3 \sum\!\!\!\!\!\!\int_{\Gamma} \; |\langle\Omega|j(0)|\Gamma\rangle|^2\delta(p-p_\Gamma), \qquad (3.1)$$

where $s = p^2$ and the integration ranges over the phase space of the hadronic system with momentum p_Γ. In the case of the strangeness-changing scalar current, the lowest lying state which contributes in the sum is the $K\pi$ system in an S-wave isospin-1/2 state.

Including also the $K\eta$ and $K\eta'$ states, the scalar spectral function can be written as

$$\begin{aligned}\rho_{\rm ph}(s) = \frac{3\Delta_{K\pi}^2}{32\pi^2}&[\sigma_{K\pi}(s)|F_{K\pi}(s)|^2+\sigma_{K\eta}(s)|F_{K\eta}(s)|^2\\ &+\sigma_{K\eta'}(s)|F_{K\eta'}(s)|^2], \qquad (3.2)\end{aligned}$$

with $\Delta_{K\pi} \equiv M_K^2 - M_\pi^2$. The two-particle phase space factors $\sigma_{KP}(s)$ take the form

$$\begin{aligned}\sigma_{KP}(s) &= \theta(s-(M_K+M_P)^2) \qquad (3.3)\\ &\times\sqrt{\left(1-\frac{(M_K+M_P)^2}{s}\right)\left(1-\frac{(M_K-M_P)^2}{s}\right)},\end{aligned}$$

where P corresponds to one of the states π, η or η', and the strangeness-changing scalar form factors $F_{KP}(s)$ are defined by

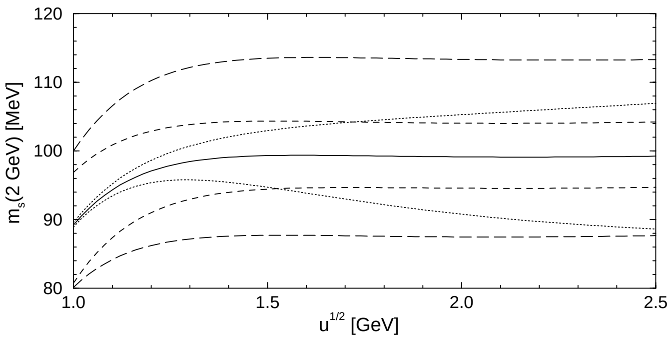

Fig. 1. The strange mass $m_s(2\,\mathrm{GeV})$ as a function of $u^{1/2}$. *Solid line:* central parameters; *long-dashed lines:* (6.10K4) with $F_{K\pi}(\Delta_{K\pi}) = 1.23$ (*upper line*), (6.10K3) with $F_{K\pi}(\Delta_{K\pi}) = 1.21$ (*lower line*); *dashed lines:* $\alpha_s(M_Z) = 0.1205$ (*lower line*), $\alpha_s(M_Z) = 0.1165$ (*upper line*); *dotted lines:* $s_0 = 4.2\,\mathrm{GeV}^2$ (*upper line*), $s_0 = 5.8\,\mathrm{GeV}^2$ (*lower line*)

$$\langle \Omega|\partial^\mu(\bar{s}\gamma_\mu u)(0)|KP\rangle \equiv -\mathrm{i}\sqrt{\frac{3}{2}}\,\Delta_{K\pi}F_{KP}(s). \tag{3.4}$$

Experimentally, it has been shown that the S-wave isospin-1/2 $K\pi$ system is elastic below roughly 1.3 GeV, and below 2 GeV, $K\eta'$ is the dominant inelastic channel [44,45]. Thus including these two states should give a good description of the scalar spectral function below 2 GeV. For completeness, however, in (3.2) we have also taken into account the $K\eta$ state. Multiparticle states, the lightest of which is the $|K\pi\pi\pi\rangle$ state, have been neglected in (3.2). Theoretically, their contributions are suppressed both in the chiral and large-N_c expansions. Nevertheless, since at an energy around 2 GeV they should play some role, we intend to investigate these contributions in the future. Owing to the positivity of the scalar spectral function, these additional contributions should slightly increase the value of the strange quark mass.

In our previous work [1], the scalar form factors $F_{KP}(s)$ have been determined for the first time from a dispersive coupled-channel analysis of the $K\pi$ system. As an input in the dispersion integrals, S-wave KP scattering amplitudes were used which had been extracted from fits to the $K\pi$ scattering data [44,45] in the framework of unitarised χPT with explicit inclusion of resonance fields [34]. The fact that the $K\eta$ channel only gives a negligible contribution to the hadronic spectral function was also corroborated in [1]. Therefore, making use of the results of [1], we are in a position to provide the scalar spectral function in an energy range from threshold up to about 2 GeV.

4 Numerical analysis

Evaluating the sum rule of (1.4) with the theoretical two-point function of Sect. 2 and the hadronic spectral function of Sect. 3, the resulting values for the running strange quark mass $m_s(2\,\mathrm{GeV})$ as a function of $u^{1/2}$ are displayed in Fig. 1. The solid line corresponds to central values for all input parameters and constitutes our main result. For $\hat{m}$, we have used $\hat{m}(2\,\mathrm{GeV}) = 4.05\,\mathrm{MeV}$ which arises from

Table 1. Values of the main input parameters and corresponding uncertainties for $m_s(2\,\mathrm{GeV})$. For a detailed explanation see the discussion in the text

Parameter	Value	Δm_s [MeV]
$\rho_{\mathrm{ph}}^{(6.10\mathrm{K4})}(s)$	$F_{K\pi}(\Delta_{K\pi}) = 1.23$	$+14.3$
$\rho_{\mathrm{ph}}^{(6.10\mathrm{K3})}(s)$	$F_{K\pi}(\Delta_{K\pi}) = 1.21$	-11.6
$\alpha_s(M_Z)$	0.1185 ± 0.0020	$^{+5.0}_{-4.7}$
$\mathcal{O}(\alpha_s^3)$	no $\mathcal{O}(\alpha_s^3)$ $2\times\mathcal{O}(\alpha_s^3)$	$+3.3$ -3.6
s_0	4.2–5.8 GeV^2	$^{+4.3}_{-3.5}$

our analysis of the next section. From the region of maximal stability of the sum rule (the extremum) which lies in the region of the $K_0^*(1430)$ resonance, we extract our central value for the strange mass $m_s(2\,\mathrm{GeV}) = 99.4\,\mathrm{MeV}$. In the stability region, the continuum is only about 25% of the full left-hand side of (1.4), so that it should be under control. To give an estimation of the uncertainties for m_s, let us discuss the inputs and their variation in more detail.

The dominant source of uncertainty for m_s is the hadronic spectral function. To obtain an estimate of the corresponding error, we have calculated m_s from different fits for the scalar form factors of [1]. Since the $K\eta$ channel was found to be unimportant, we have only considered the two-channel spectral functions with contributions from $K\pi$ and $K\eta'$. As the fits for the form factors, we utilise here our best fits (6.10K3) and (6.10K4) of [1][2]. As was discussed in detail in [1], however, these fits are not unique, but can be parametrised by $F_{K\pi}(\Delta_{K\pi})$ which should take the value 1.22 ± 0.01. The solid line in Fig. 1 then corresponds to the central value $F_{K\pi}(\Delta_{K\pi}) = 1.22$ and an average of the spectral functions for (6.10K3) and (6.10K4). Varying $F_{K\pi}(\Delta_{K\pi})$ for both fits, the largest m_s is obtained for (6.10K4) with $F_{K\pi}(\Delta_{K\pi}) = 1.23$ and the smallest for (6.10K3) with $F_{K\pi}(\Delta_{K\pi}) = 1.21$. Both cases are displayed as the long-dashed lines in Fig. 1 and the variation of m_s has been collected in Table 1.

The next-largest uncertainty for m_s which is related to the perturbative expansion results from two sources. On the one hand there is an error on the input value for α_s and on the other hand, there are unknown higher order corrections. For the strong coupling, we have used the PDG value [46] and varied α_s within its error. The corresponding variation of m_s is shown as the dashed line in Fig. 1 where the upper line is the case with $\alpha_s(M_Z) = 0.1165$ and the lower line with $\alpha_s(M_Z) = 0.1205$. To estimate the second uncertainty, we have either completely removed the $\mathcal{O}(\alpha_s^3)$ correction or doubled its value. The resulting errors for m_s are presented in Table 1.

Another uncertainty for m_s results from a variation of the continuum threshold s_0. Our central value $s_0 = 4.75\,\mathrm{GeV}^2$ has been chosen such as to obtain a maximal stability of the sum rule in the region of interest. As our range for s_0 we have chosen $s_0 = 4.2$–$5.8\,\mathrm{GeV}^2$. The lower

[2] The strange mass resulting from the fit (6.11K4) of [1] is practically identical to m_s from the fit (6.10K4). Thus, for this work, we have not considered this fit separately

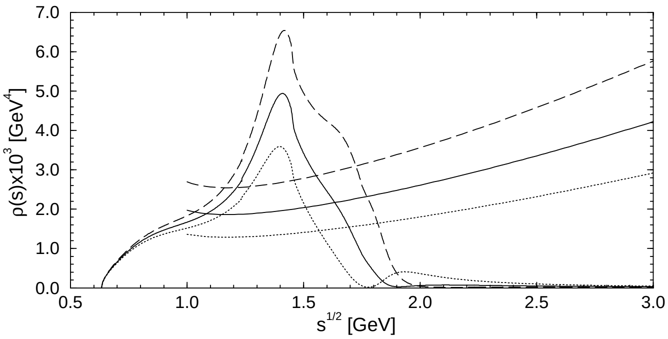

Fig. 2. The theoretical as well as phenomenological spectral functions used in our m_s determination. Solid lines: central spectral functions; long-dashed lines: $\rho_{\rm ph}(s)$ for (6.10K4) with $F_{K\pi}(\Delta_{K\pi}) = 1.23$ and $\rho_{\rm th}(s)$ for $m_s = 115\,$MeV; dotted lines: $\rho_{\rm ph}(s)$ for (6.10K3) with $F_{K\pi}(\Delta_{K\pi}) = 1.21$ and $\rho_{\rm th}(s)$ for $m_s = 83\,$MeV

value already lies close to the region of the $K_0^*(1950)$ resonance and around the higher value the third scalar resonance would be expected from Regge phenomenology. Thus the chosen range should be rather conservative and it is gratifying that the most stable sum rule is reached for an s_0 within this range. The dotted lines in Fig. 1 show the corresponding variation of m_s with $s_0 = 4.2\,\text{GeV}^2$ (upper line) and $s_0 = 5.8\,\text{GeV}^2$ (lower line). Again, the error on m_s from the variation of s_0 is listed in Table 1. Because there is no stability for $s_0 = 4.2\,\text{GeV}^2$, as the relevant value we have taken m_s in the region around 1.6 GeV, where stability occurs for the central parameters. For $u^{1/2} \geq 2\,$GeV, the continuum is larger than 50% of the lhs of (1.4), and there the sum rule becomes unreliable.

Except for the quark condensate, which will be discussed in the next section, the values of the condensate parameters have been taken according to [9]. However, as already stressed above, their relevance for the m_s determination is negligible and thus also the corresponding uncertainty. Instanton contributions to the scalar and pseudoscalar two-point functions have been considered in [47–51]. In the framework of the instanton-liquid-model [52], in [16] it was shown that the prediction for m_s from scalar Borel sum rules is only lowered by 2 MeV. In view of the uncertainties from other sources, we have therefore neglected instanton contributions.

Adding the errors of Table 1 in quadrature, we arrive at our final result for the strange quark mass:

$$m_s(2\,\text{GeV}) = 99.4^{+16.1}_{-13.5}\,\text{MeV} = 99 \pm 16\,\text{MeV}. \qquad (4.1)$$

To be more conservative, we have taken the larger of the errors as our final uncertainty for the strange quark mass.

In [9,13] the strange mass was also calculated from the first moment sum rule which arises by differentiating (1.4) with respect to u. Performing this exercise here, we find that the resulting sum rule is less stable and the region of maximal stability is lowered to about 1 GeV, where perturbative as well as power corrections are more important. Nevertheless, for our central value of s_0, m_s only decreases by less than 3 MeV, and if a lower s_0 is chosen to get a more stable sum rule, the resulting value for m_s is in complete agreement with (4.1), providing additional support to our result.

To conclude this section, in Fig. 2, we display a comparison of the theoretical as well as phenomenological spectral functions used in our m_s determination. The solid lines correspond to central spectral functions. The long-dashed lines show $\rho_{\rm ph}(s)$ and $\rho_{\rm th}(s)$ corresponding to the largest value of m_s, and the dotted lines to the smallest. In the $K_0^*(1430)$ resonance region, the hadronic spectral functions differ by almost a factor of two. Thus, if it would become possible to experimentally measure the scalar spectral function or $F_{K\pi}(s)$ in this region with smaller uncertainties, the strange mass determination from scalar sum rules could still be improved.

5 Conclusions

Let us now come to a comparison of our result (4.1) for the strange quark mass with other recent determinations of this quantity. A related approach to the one followed here, also using the scalar sum rule, has been applied in [12], where $m_s(2\,\text{GeV}) = 107 \pm 13\,\text{MeV}$ was obtained. In this work, however, the hadronic spectral function $\rho_{\rm ph}(s)$ was estimated from the single-channel Omnès form factor $F_{K\pi}^{\text{Omnès}}(s)$. In view of our discussion about the dependence of the scalar $K\pi$ form factor on the parametrisation of the corresponding S-wave $I = 1/2$ phase shift in the elastic, single-channel case [1], the error in [12] appears underestimated, although the central values are in good agreement.

The older scalar sum rule analyses of [9–11], on the other hand, have parametrised the phenomenological spectral function with a Breit–Wigner Ansatz which was normalised to the scalar form factor at the $K\pi$ production threshold. As was discussed in detail in [14,12], this parametrisation overestimates the scalar spectral function, because in the scalar channel the resonance contribution interferes destructively with the large non-resonant background. Therefore, the resulting strange mass values turned out larger than our central result presented here, and should be discarded in the future. Nevertheless, within the uncertainties at the time, including $\mathcal{O}(\alpha_s^3)$ corrections the result $m_s(2\,\text{GeV}) = 130\,\text{MeV}$ [9] still was compatible with our present finding of (4.1).

Very recently, the determination of the strange mass from pseudoscalar finite energy sum rules was reanalysed in [16]. In this case, instanton contributions play some role and have to be included. The resulting value $m_s(2\,\text{GeV}) = 100 \pm 12\,\text{MeV}$ then is in perfect agreement to our finding of (4.1). The status of the extraction of m_s from the hadronic e^+e^- cross section is less clear. Whereas [27] finds a value of $m_s(2\,\text{GeV}) = 129 \pm 24\,\text{MeV}$, in [25] it is pointed out that large isospin breaking corrections significantly lower the result for the strange mass to about $m_s(2\,\text{GeV}) = 95\,\text{MeV}$ and yield considerably larger uncertainties of the order of 45 MeV. We therefore conclude that further work in this channel is needed, before a definite conclusion can be reached.

The most recent determination of m_s from the Cabibbo suppressed τ-decay width gave $m_s(2\,\mathrm{GeV}) = 116^{+20}_{-25}$ MeV [23], in agreement with (4.1) within the quoted error bars, although yielding a somewhat larger central value. In addition to experimental uncertainties and a sizeable sensitivity to the quark-mixing parameter V_{us}[3], the precision of the τ-decay value is limited by the bad perturbative behaviour of the $J = 0$ contribution. Our determination of the scalar spectral function could be used to disentangle the $J = 0$ and $J = 1$ components of the τ data, allowing for a more accurate determination of m_s from the theoretically well behaved $J = 1$ contribution. In any case, whereas the dominant uncertainty for m_s from scalar sum rules arises from the phenomenological part, in the τ decays it is due to the perturbative expansion, and in this sense both determinations can be considered as complementary.

Two recent reviews of determinations of the strange quark mass from lattice QCD have been presented in [32, 33], with the conclusions $m_s(2\,\mathrm{GeV}) = 110 \pm 25$ MeV and $m_s(2\,\mathrm{GeV}) = 120\pm25$ MeV respectively. The error in these results is dominated by the uncertainty resulting from dynamical fermions, whereas the calculations of m_s in the quenched theory, based for example on the kaon mass, are already very precise. Generally, in unquenched calculations the strange mass is found below 100 MeV. Nevertheless, the agreement between lattice QCD and QCD sum rule determinations of m_s is already very satisfactory.

Chiral perturbation theory provides rather precise information on ratios of the light quark masses. Two particular ratios are [55]

$$R \equiv \frac{m_s}{\hat{m}} = 24.4 \pm 1.5,$$
$$Q^2 \equiv \frac{(m_s^2 - \hat{m}^2)}{(m_d^2 - m_u^2)} = (22.7 \pm 0.8)^2. \tag{5.1}$$

From these ratios, one further deduces $m_u/m_d = 0.551 \pm 0.049$ and $m_s/m_d = 18.9 \pm 1.3$, where the uncertainties have been estimated by assuming Gaussian distributions for the input quantities. Our central values are in agreement with the results quoted in [55], although we find somewhat larger errors. The ratio m_u/m_d has also been calculated in [56] with the result $m_u/m_d = 0.46 \pm 0.09$. Within the uncertainties, this ratio is compatible with the previous one. Using the former ratios, together with our result (4.1) for m_s, we obtain for m_u and m_d

$$m_u(2\,\mathrm{GeV}) = 2.9 \pm 0.6\,\mathrm{MeV},$$
$$m_d(2\,\mathrm{GeV}) = 5.2 \pm 0.9\,\mathrm{MeV}. \tag{5.2}$$

The resulting value for the sum of up and down quark masses, $(m_u + m_d)(2\,\mathrm{GeV}) = 8.1 \pm 1.4$ MeV is compatible with the finding $(m_u + m_d)(2\,\mathrm{GeV}) = 9.6 \pm 1.9$ MeV [57,58], and in good agreement with the result $(m_u + m_d)(2\,\mathrm{GeV}) = 7.8 \pm 1.1$ MeV [16], both obtained with finite energy sum rules for the pseudoscalar channel.

The knowledge of the light quark masses also allows for a determination of the light quark condensate from the Gell-Mann–Oakes–Renner relation [59]:

$$(m_u + m_d)\langle \Omega|\bar{q}q|\Omega\rangle = -f_\pi^2 M_\pi^2 (1 - \delta_\pi). \tag{5.3}$$

The term δ_π summarises higher order corrections in the chiral expansion and also contains the renormalisation dependence mentioned in the introduction [38]. Using a generous range $\delta_\pi = 0.05\pm0.05$ for this quantity [38], together with the quark masses of (5.2) as well as $f_\pi = 92.4$ MeV and $M_\pi = 138$ MeV, we arrive at

$$\langle \Omega|\bar{q}q|\Omega\rangle(2\,\mathrm{GeV}) = -(267 \pm 17\,\mathrm{MeV})^3, \tag{5.4}$$

which can be considered as an update of previous determinations of the light quark condensate $\langle\bar{q}q\rangle$. Since it is still more common to quote the quark condensate at a scale of 1 GeV we also provide the corresponding value: $\langle\bar{q}q\rangle(1\,\mathrm{GeV}) = -(242 \pm 16\,\mathrm{MeV})^3$. This value can be compared with direct determinations of the quark condensate from QCD sum rules [30].

To conclude, in this work we have determined the masses of the light up, down and strange quarks. To this end, first the strange mass m_s was evaluated in the framework of QCD sum rules for the scalar correlator with the result (4.1). Our work improves previous analyses of this system by calculating the phenomenological spectral function which enters the sum rule through a dispersive coupled-channel analysis of the contributing hadronic states, making use of our recent work [1] on strangeness-changing scalar form factors. The masses of the up and down quarks m_u and m_d were then calculated employing ratios of quark masses known from χPT, together with our result (4.1) for m_s. Our final values for m_u and m_d have been presented in (5.2).

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[3] The reader should note that a slightly lower central value for m_s is obtained if the value $|V_{us}| = 0.2207$ [53,54], and not the unitarity-constraint fit $|V_{us}| = 0.2225$ [46], is used in the τ sum rule

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